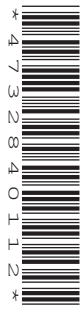
OCR
RECOGNISING ACHIEVEMENT

Friday 20 January 2012 – Afternoon

AS GCE MATHEMATICS

4725 Further Pure Mathematics 1

QUESTION PAPER



Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The complex number $a + 5\mathrm{i}$, where a is positive, is denoted by z. Given that $|z| = 13$, find the value of a and hence find $\arg z$. **[4]**

2 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 2 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & 6 \\ 3 & -5 \end{pmatrix}$, and **I** is the 2×2 identity matrix. Given that $p\mathbf{A} + q\mathbf{B} = \mathbf{I}$, find the values of the constants p and q. **[5]**

3 Use an algebraic method to find the square roots of $3 + (6\sqrt{2})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact real numbers. **[6]**

4 Find $\sum_{r=1}^{n} r(r^2 - 3)$, expressing your answer in a fully factorised form. **[6]**

5 **(a)** Find the matrix that represents a reflection in the line $y = -x$. **[2]**

(b) The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$.

(i) Describe fully the geometrical transformation represented by **C**. **[2]**

(ii) State the value of the determinant of **C** and describe briefly how this value relates to the transformation represented by **C**. **[2]**

6 Sketch, on a single Argand diagram, the loci given by $|z - \sqrt{3} - \mathrm{i}| = 2$ and $\arg z = \frac{1}{6}\pi$. **[6]**

7 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$.

(i) Show that $\mathbf{M}^4 = \begin{pmatrix} 81 & 0 \\ 80 & 1 \end{pmatrix}$. **[3]**

(ii) Hence suggest a suitable form for the matrix $\mathbf{M}^n$, where n is a positive integer. **[2]**

(iii) Use induction to prove that your answer to part **(ii)** is correct. **[4]**

8 **(i)** Show that $\frac{r}{r+1} - \frac{r-1}{r} \equiv \frac{1}{r(r+1)}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots + \frac{1}{n(n+1)}.$$ **[4]**

(iii) Hence find $\sum_{r=n+1}^{\infty} \frac{1}{r(r+1)}$. **[2]**

9 The matrix $\mathbf{X}$ is given by $\mathbf{X} = \begin{pmatrix} a & 2 & 9 \\ 2 & a & 3 \\ 1 & 0 & -1 \end{pmatrix}$.

(i) Find the determinant of $\mathbf{X}$ in terms of a. **[3]**

(ii) Hence find the values of a for which $\mathbf{X}$ is singular. **[3]**

(iii) Given that $\mathbf{X}$ is non-singular, find $\mathbf{X}^{-1}$ in terms of a. **[4]**

10 The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots α, β and γ.

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. **[3]**

The cubic equation $x^3 + ax^2 + bx + c = 0$ has roots α^2, β^2 and γ^2.

(ii) Show that $c = -\frac{4}{9}$ and find the values of a and b. **[9]**

Question	Answer	Marks	Guidance
1	$a^2 + 5^2 = 13^2$ $a = 12$ $\tan^{-1}\frac{5}{a}$ 0.395 or 22.6° or 0.126π	M1 A1 M1 A1FT **[4]**	Use formula for modulus Obtain correct answer Use formula for argument Obtain correct answer allow 0.39
2	$3p + 4q = 1, \quad -3p - 5q = 1, \quad 2p + 3q = 0$ $p = 3$ and $q = -2$	B1 M1 A1 M1 A1 **[5]**	State identity matrix is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Find a pair of simultaneous equations Correct pair of distinct equations Attempt to solve Obtain correct answers
3	$x^2 - y^2 = 3$ and $xy = 3\sqrt{2}$ $x^4 - 3x^2 - 18 = 0$ or $y^4 + 3y^2 - 18 = 0$ $x = \pm\sqrt{6}$ or $y = \pm\sqrt{3}$ $\pm(\sqrt{6} + i\sqrt{3})$	M1 A1 M1 M1 A1 A1 **[6]**	Attempt to equate real and imaginary parts Obtain both results Eliminate to obtain quadratic in x^2 or y^2 Solve to obtain x or y value Both values correct Correct answers as complex numbers

Question			Answer	Marks	Guidance	
4			$\frac{1}{4}n^2(n+1)^2 - \frac{3}{2}n(n+1)$ $\frac{1}{4}n(n+1)(n+3)(n-2)$	M1 DM1 A1 M1 A1 A1 **[6]**	Express as difference of two series Use standard series results Obtain correct unsimplified answer Attempt to factorise At least factor of $n(n+1)$ Obtain correct answer	From their unsimplified answer
5	(a)		$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1 B1 **[2]**	Each column correct	
5	(b)	(i)		B1 DB1 **[2]**	Stretch Scale factor 4 in the y direction	Not "in the y-axis"
5	(b)	(ii)	4	B1 B1 **[2]**	Correct value of determinant Scale factor for area	Allow scale factor of stretch or eqiv.
6				B1 B1 B1 B1 B1 B1 **[6]**	Circle Centre $(\sqrt{3},1)$ Passing through O and crosses y-axis again Line, with correct slope shown $\frac{1}{2}$ line starting at O Completely correct diagram for both loci	Ignore shading

Question			Answer	Marks	Guidance	
7	(i)			M1 A1 A1 **[3]**	Attempt at matrix multiplication Obtain $\mathbf{M}^2$ correctly Obtain **given answer** correctly	
7	(ii)		$\begin{pmatrix} 3^n & 0 \\ 3^n - 1 & 1 \end{pmatrix}$	B1 B1 **[2]**	3 elements correct 4th element correct	
7	(iii)		$\begin{pmatrix} 3^{k+1} & 0 \\ 3^{k+1} - 1 & 1 \end{pmatrix}$	B1 M1 A1 B1 **[4]**	Show that their result is true for $n = 1$ or 2 Attempt to find $\mathbf{M}^k \cdot \mathbf{M}$ or vice versa Obtain correct answer Complete statement of induction conclusion	 Must have 1st 3 marks
8	(i)			M1 A1 **[2]**	Combine with a common denominator Obtain **given** answer correctly	
8	(ii)		$\frac{n}{n+1}$	M1 A1 M1 A1 **[4]**	Express terms using **(i)** At least 1st two and last two correct Show terms cancelling Obtain correct answer, in terms of n	

Question			Answer	Marks	Guidance	
8	(iii)		$1-\frac{n}{n+1}$	B1 B1FT **[2]**	$\lim_{n\to\infty}\frac{n}{n+1}=1$ This value – **(ii)**	
9	(i)		det **X** = $\Delta = 10-9a-a^2$	M1 M1 A1 **[3]**	Show correct expansion process for 3×3 Correct evaluation of any 2×2 Obtain correct answer aef	
9	(ii)		$a = 1$ or -10	M1 A1FT A1FT **[3]**	Their det **X** = 0 Obtain correct answers from their **(i)**	
9	(iii)		$\frac{1}{\Delta}\begin{pmatrix} -a & 2 & 6-9a \\ 5 & -a-9 & 18-3a \\ -a & 2 & a^2-4 \end{pmatrix}$	M1 A1 A1 B1ft **[4]**	Show correct process for adjoint entries Obtain at least four correct entries in adjoint Obtain completely correct adjoint Divide by their determinant	
10	(i)		$\alpha+\beta+\gamma=3$ $\alpha\beta+\beta\gamma+\gamma\alpha=2$ $\alpha\beta\gamma=-\frac{2}{3}$	B1 B1 B1 **[3]**	State correct value State correct value State correct value	

Question			Answer	Marks	Guidance	
10	(ii)		EITHER			
				M1	$c = (\pm)\alpha^2\beta^2\gamma^2$	
			$c = -\frac{4}{9}$	A1FT	Obtain **given** correct answer	FT for sign error in **(i)**
			$\sum\alpha^2 = (\sum\alpha)^2 - 2\sum\alpha\beta$	M1	Use correct expression	
			5	A1FT	Obtain correct value	FT for sign error in **(i)**
			$a = -5$	A1FT	Obtain answer correctly	Sign change done correctly
				M1*	Attempt to find an identity	
			$\sum\alpha^2\beta^2 = (\sum\alpha\beta)^2 - 2\alpha\beta\gamma\sum\alpha$	A1	Obtain correct identity	
				DM1	Use appropriate values	
			$b = 8$	A1	Obtain correct answer cao	
				[9]		
			OR			
				B1	State or use correct substitution	
				M1	Rearrange, fractional indices isolated	
				DM1	Square both sides	
				DM1	Expand and simplify	
			$9y^3 - 45y^2 + 72y - 4 = 0$	A1	Obtain correct equation	
				M1	Use coefficients of their cubic	
			$c = -\frac{4}{9}$	A1	Obtain given answer correctly	
			$a = -5$	A1FT	Obtain correct answer	
			$b = 8$	A1FT	Obtain correct answer	
					SC mixture of methods only A1FT for a and b	
				[9]		